ACCOUNTING FOR NONLINEAR LOSSES WHICH OCCUR WHILE AN ELECTRODE IS HEATED BY A SHIFTING ARC DISCHARGE

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UDC 537.523.5:536.24

The heating of an electrode by a shifting arc discharge in vacuum is analyzed here. Heat losses are accounted for quasidynamically as well as directly in the boundary condition for the differential equation.

The temperature distribution in a solid body heated from point sources is of considerable interest in diverse branches of thermophysics and its engineering applications. The thermal state of an electrode affected by an electric arc discharge determines to a large extent the emission process which produces charged particles necessary to sustain the arc, and also the process of electrode wear. A study of the temperature fields at the surface of a treated material makes it possible to evaluate the effectiveness of the thermal energy transmitted by electron beams.

Generally, determining the temperature function with respect to the field coordinates reduces to solving the equation of heat conduction by either the point-source or the distributed-source method [1, 2] – depending on the magnitude of the heat concentration factor. However, such solutions do not account for thermal energy losses at the surface of the heated body which are incurred primarily by evaporation of the material and by radiation according to the Stefan–Boltzmann law.

The heat sources method yields the following expression for the temperature as a function of space coordinates and of the time of action of a heat source [2]:

$$T(x,\ y,\ z,\ t)=\frac{2q}{c\gamma(4\pi a)^{3/2}}\ \exp\left(-\frac{vx}{2a}\right)\int_0^t\frac{dt'}{\sqrt{t'}\,(t_0+t')}\ \exp\left[-\frac{z^2}{4at'}-\frac{r_0^2}{4a(t_0+t')}-\frac{v^2(t_0+t')}{4a}\right], \tag{1}$$

$$t_0=\frac{1}{4ak},\quad k=\frac{1}{r_0^2}; \tag{2}$$

$$r_0=\sqrt{\frac{I}{\pi i}}. \tag{3}$$

In the case of an electric discharge we find the magnitude of r_0 from Eq. (3). The values of integral (1) has been determined numerically with the aid of a computer. The calculations were performed for tungsten [3].

Nonlinear heat losses can, to a first approximation, be accounted for as follows.

As is well known, a body at some initial temperature will warm up when exposed to a heat source and its temperature will rise until the heat from the source lost in evaporation according to the Langmuir law

$$G=10^{7.5-\frac{4.03\cdot 10^4}{T}} \tag{4}$$

and in radiation according to the Stefan–Boltzmann law

$$W=\sigma T^4 \tag{5}$$

begins to become significant.

V. I. Lenin All-Union Electrical Engineering Institute. Institute of High Temperatures, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 19, No. 4, pp. 678-681, October, 1970. Original article submitted September 30, 1969.

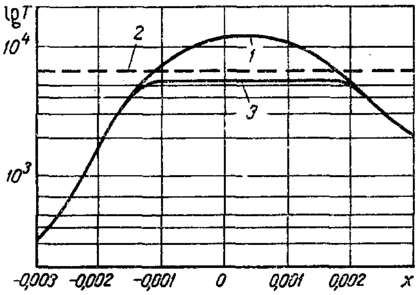

Fig. 1. Temperature, °K as a function of the coordinate, m: 1) temperature distribution not accounting for heat source losses; 2) quasidynamic allowance for heat losses (solution to the heat balance equation); 3) temperature distribution accounting for the heat source losses.

At the same time, of course, there is also heat lost on phase and structural transformations, on ionization, etc. Calculations have shown, however, that these losses are so small as to be negligible [3] compared to the large power densities produced at the electrodes of an arc in vacuum – power densities of the order of 10^9-10^{12} W/m^2. As the heat energy supplied to the body increases, the heat losses begin to increase too and a point is reached, at a certain power density level of the source, when all the power received by the body surface will be expended on these nonlinear losses. Such a state of a heated body will be characterized by some temperature ceiling which depends on the given power density of the constant-temperature source and which is determined by the saturation process [2]. This temperature is easily found from the heat balance equation set up for a specific source power density, which in the case of an electric arc in vacuum is determined essentially by the arc current. It is assumed here that, when the arc is much shorter than the electrode diameter, the energy released by an arc in vacuum distributes equally between the electrodes [6]. This assumption has been supported by experimental data [4] on:

$$\frac{g' i u_g}{2} = k_1 \operatorname{grad} T + \lambda G + \sigma T^4. \tag{6}$$

In order to account for the heat losses while solving Eq. (1), we may limit the entire temperature distribution based on Eq. (1) by the steady-state temperature particular for any given source power density and found from the heat balance equation in [6].

This way of accounting for nonlinear heat losses has the disadvantage, however, that it does not provide any information about the temperature field distortions near the temperature ceiling and, therefore, the temperature distribution below the ceiling remains unknown.

In order to determine the temperature field distortion near the temperature ceiling, the temperature distribution was derived from the fundamental equation of heat conduction with the nonlinear losses accounted for directly.

Such a problem reduces to the following mathematical formulation. It is necessary to solve the equation:

$$\frac{\partial T}{\partial t} = a\nabla^2 T \tag{7}$$

with the boundary conditions:

$$b(T) = \left.\frac{\partial T}{\partial z}\right|_{z=0} = -\left[\alpha \exp\{-k[(x-vt)^2 + y^2]\} - \lambda G - \sigma T^4\right],$$

$$T|_{x=l} = T|_{y=b} = T|_{z=d} = 0 \tag{8}$$

and the initial conditions

$$T(x, y, z, 0) = 0, \tag{9}$$

where

$$\alpha = \frac{iu_A}{2k_1}; \quad k = \frac{\pi i}{I}. \tag{10}$$

The temperature distribution was calculated accordingly on a computer by a numerical method based on approximating the sought function by a variable-step array of differences [5].

The temperature field determined in this way for a current density of 10^8 A/m^2 with a heat source shifting at a velocity of 0.1 m/sec and active for $5 \cdot 10^{-3}$ sec has shown that the steady-state temperature level coincides with the temperature ceiling calculated from the heat balance equation, while the temperature field distortion below the ceiling is insignificant (Fig. 1) and may be attributed to the error of the difference method.

Consequently, a solution of Eq. (1) with quasidynamic allowance for nonlinear heat losses yields the correct temperature distribution below the steady-state temperature level.

NOTATION

x, y, z	are the space coordinates, m;
t	is the time, sec;
γ	is the density, kg/m^3;
k	is the heat concentration factor;
t_0	is the parameter characterizing the heat concentration factor of the source;
r_0	is the nominal radius of heat source, m;
I	is the electric current, A;
i	is the current density, A/m^2;
g'	is the thermal equivalent;
v	is the velocity of shifting heat source, m/sec;
q	is the incident heat flux density, W/m^2;
T	is the temperature, °K;
σ	is the radiation coefficient;
u_A	is the voltage drop along the arc, V;
k	is the thermal conductivity;
$r^2 = x^2 + y^2$.	

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